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Granular gas in a periodic lattice

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Abstract

Glass beads are placed in the compartments of a horizontal square grid. This grid is then vertically shaken. According to the reduced acceleration Γ of the system, the granular material exhibits various behaviours. By counting the number of beads in each compartment after shaking, it is possible to define three regimes. At low accelerations, the grains remain in their compartment, and the system is frozen. For very large accelerations, the grains bounce out of the compartments and behave as a 'binomial gas': the system is homogeneous. For intermediate accelerations, grains form clusters, i.e. grains gather in some particular compartments. In that regime, the probability for a bead to escape from a site depends on the number of beads contained in the concerned compartment. The escape probability has been measured with respect to the number of beads in a compartment. Above a given number of beads, the beads remain trapped in the compartment. A basic numerical model reproduces some of the results and allows us to explore the dependence on the initial conditions.

(Some figures in this article are in colour only in the electronic version)

1. Introduction

A granular material exhibits hybrid properties that resemble the properties of a solid, a liquid or a gas according to the external constraints [1]. The granular 'gas' state is obtained by shaking a box containing grains, e.g. spherical beads. When the amplitude of the excitation is large, the grains bounce in any direction, giving the apparent properties of a gas. However, the gas is made of hard spheres which may rotate and collisions are not perfect. The system is dissipative, because the total energy after a collision is lower than the energy before. These simple additional ingredients confer to the granular gas state particular behaviours.

An experiment that concerns granular materials was reported in a vulgarization article by Schlichting and Nordmeier in 1996 (it is not clear who discovered this phenomenon) [2]. This experiment consists of a box that is compartmented by a central vertical wall that allows exchange at the top. Initially, the same number of beads (millimetre size) is set in each compartment. When the vertical amplitude shaking is low, the grains remain in their compartment. The amplitude is increased to allow grains to jump from one compartment to another one. From a perfect gas point of view, the number of particles is roughly the same in both compartments and some fluctuations can be considered. However, spontaneously, the grains eventually gather in the same compartment. We observe a right–left symmetry breaking. A basic and naïve interpretation is that entropy spontaneously decreases: the granular Maxwell's demon was awakened. Actually, the apparent paradox is attributed to the inelastic collisions of the beads. More exactly, the system is nonconservative which cannot therefore be compared to the physics of an equilibrium state. A highly shaken granular material is not a gas [4]. The coefficient of restitution, defined as the ratio between the total energy after and before the impact, is lower than unity. After the impact, the grains have less energy and consequently are not able to go as far as before the impact. The situation is amplified when one bead hits an assembly of grains. Indeed, because of the numerous successive impacts, the energy dissipates very quickly. The grain remains *stuck* in the cluster. Finally, when the amplitude of the vibration is very large, the number of grains in each compartment balances again. To sum up, according to the shaking amplitude, three 'states' have been identified: the frozen state (beads remain in the same compartment), the clustering state, and the gas-like state.

The experiment was revisited in 1999 by Eggers who found a theoretical argument founded on hydrodynamics to explain the phenomenon [5, 6]. In 2001, Lohse's group extended to several aligned compartments experimentally [7] and theoretically [8]. Bifurcations and transitions have been carefully analysed and extended to several compartments in 2006 [9] and 2007 [15]. The gravity influence has been experimentally analysed using the diamagnetic properties of the bismuth grains to control the gravity [14]. The critical acceleration of the shaker for obtaining spontaneous symmetry breaking has been measured with respect to this effective gravity. Finally, the granular Maxwell daemon has been thought to be used to segregate a mixture of two granular species [10]. The influence of the bidispersity has been revisited experimentally [11] and numerically [12, 13] in two compartments. Phase diagrams have been determined according to the acceleration of the plate and the ratio of grain species. Oscillations, i.e. regular exchanges of particles between the compartments, have been observed in these later references.

This work generalized the phenomenon by considering a grid of square compartments (figure 1, top). Looking from above, the lattice can be seen as a periodic potential that defines many traps. Energy is provided by the shaker to the particles. When the energy is sufficient compared to the potential barriers, the particles can jump from one trap to another. The measurements consist of counting the number of beads in each compartment after a vibration run. In this paper, we choose to use the maximum acceleration of the plate as the control parameter. To provide a cursory glance at the results, according to this parameter, three regimes can be observed: (i) no exchange, (ii) clustering and (iii) gas. The ability of a trap to keep the particles has been measured and allows us to establish a simple model. Indeed, we will show that for a fixed acceleration, the probability that a bead escapes from a compartment depends on the number of beads in that compartment. Using only this parameter, a simple model was built to reproduce clustering.

2. Experimental setup

On a square plate, a grid has been fixed. It is made of $N_c = 9 \times 9 = 81$ square compartments. Each compartment measures 23 mm on each side side and 30 mm high. The grains are glass beads of 6 mm diameter. The coefficient of restitution, ε, given by the ratio between the

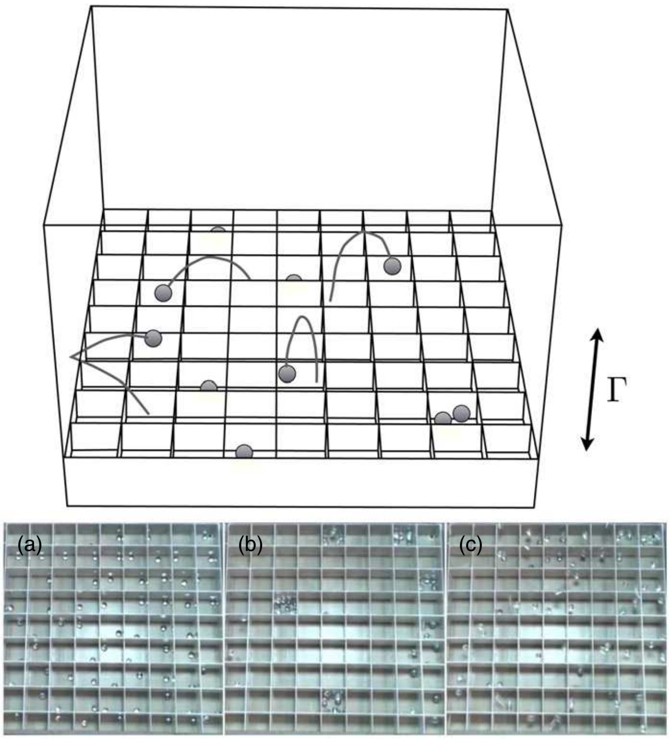

Figure 1. Top: schematic view of the 9 by 9 grid. Bottom: snapshots of the grid during a sweep in acceleration (at 25 Hz) between $\Gamma_v = 0$ to 4. (a) Initial state, one bead per compartment, (b) clustering state and (c) gas state when $\Gamma_v = 4$. A movie can be seen at the link [3].

speed just after and just before a rebound, has been measured to be $\varepsilon = 0.83$. The coefficient of restitution of a bead on a bead is rather difficult to determine. However, it is larger than ε. The grid is vertically vibrated using an electromagnetic shaker (GW 55). The motion is sinusoidal of amplitude, A, and of frequency, f, which is fixed to 25 Hz. The motion is characterized by the reduced acceleration $\Gamma_v = A(2\pi f)^2/g$ where g is the gravity. The acceleration is measured using an accelerometer that is fixed on the plate. A measurement consists of stopping the vibration and counting the number of beads, n, in each compartment after a time, T, that is assumed to be very long compared to the characteristic time between two successive collisions. This later can be estimated to be 500 μs ($\approx (nf)^{-1}$). The transient state is considered as over after 300 s. The measurements have been taken afterwards.

3. Experimental results

As the initial state, we chose to place one bead per compartment. The number, $\mathcal{N}$, of beads is therefore fixed to 81. The plate is then vibrated at 25 Hz with a fixed amplitude. After 300 s, the electromagnetic shaker is stopped. The beads are counted in each compartment. In doing so, we obtain the number, C_n, of compartments filled with n beads, i.e. the distribution of filled compartments. The experiments were carried out three times to smooth the distributions. The proportion of beads located in the compartment populated by n beads is given by $nC_n/\mathcal{N}$. It should be noted that $\sum_i \mathrm{i}C_i/\mathcal{N} = 1$. In figure 2, this quantity is represented for three reduced

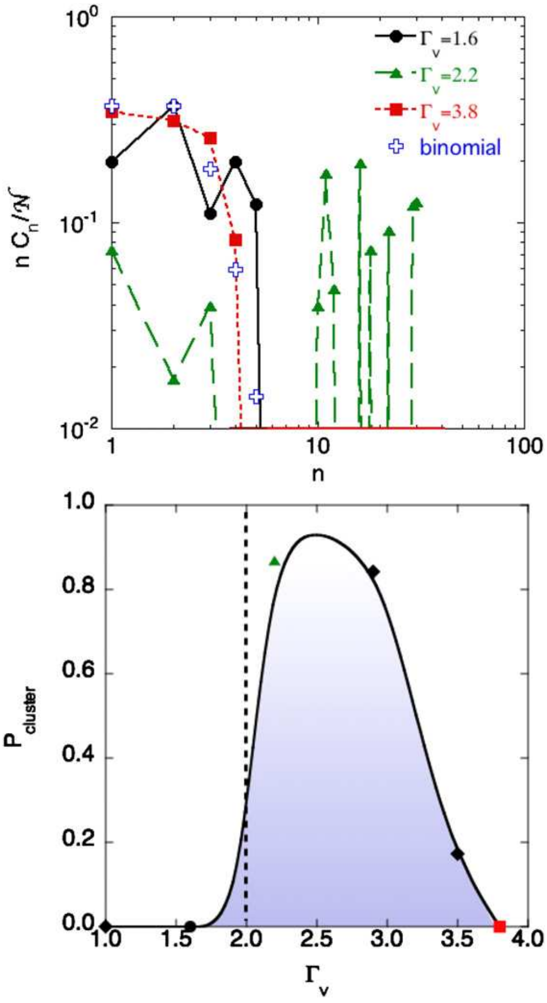

Figure 2. Top: proportion of beads located in the compartment populated by *n* beads with respect to the number of beads per compartment. Three reduced accelerations are represented: $\Gamma_v = 1.6$ (circles), 2.2 (triangles) and 3.8 (squares). A binomial distribution (equation (2)) normalized by $\mathcal{N}$ is also plotted (blue crosses). Bottom: proportion of beads considered as being trapped (namely in a compartment having more than nine beads) with respect to the acceleration Γ_v. The symbols are the same as in the figure at the top. The vertical dashed line represents the acceleration threshold $\Gamma_v \approx 2$ for a bead to escape from a compartment. This value is deduced from figure 3. The curve is only a guide.

accelerations that have been considered $\Gamma_v = 1.6$ (circles), 2.2 (triangles) and 3.8 (squares). For very low acceleration, the beads cannot but stay in the trap. Up to $\Gamma_v = 1.6$, the beads may jump high enough to escape a trap. The maximum of the distribution is located around $n \approx 2$. The dynamics are rather slow. It is not clear whether a stationary state is reached (even after 600 s). On the other hand, above $\Gamma_v = 2.2$ till 3.5, clusters are rapidly observed. When a compartment is filled with more than a given number of beads (in this case 9, see below), *n* cannot but increase. Above $\Gamma_v = 3.5$, no cluster can be observed. On the contrary, the proportion of compartments filled with one bead is the largest and monotonically decays

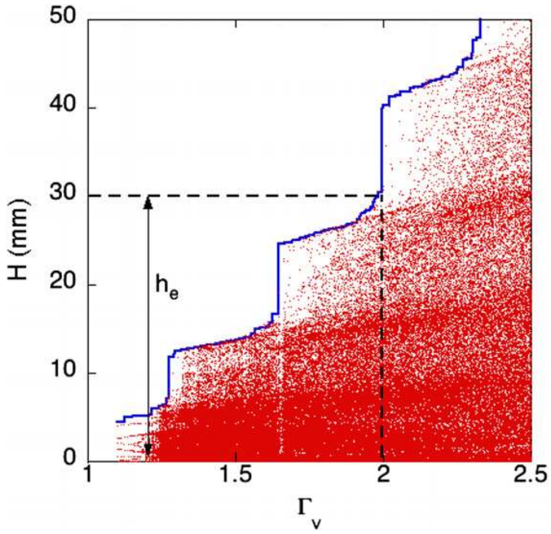

Figure 3. Heights reached by a bouncing ball with a coefficient of restitution equal to 0.83 with respect to the acceleration Γ_v at 25 Hz. The horizontal line represents the height of a compartment at 30 mm and the height for which the centre of mass of the bead may be higher than the height of a compartment. The continuous line represents the envelope curve and consequently the maximum reachable height for the bead at the considered acceleration.

with n. In figure 1 (bottom), three snapshots of the grid are shown in three particular states during an acceleration Γ_v sweep from 0 to 4. Figure 1(a) is the initial state, while figure 1(b) presents a snapshot of the cluster state, clearly visible in the middle of the box. Finally, when the acceleration is strongly increased, the beads jump easily from one compartment to another and the distribution is drastically changed to a gas-like state (figure 1(c)).

It is possible to evaluate that the acceleration required to allow one single bead to escape from a compartment depends on the coefficient of restitution of the bead on the plate. The maximum height reached by a ball bouncing on a vibrated plate may be evaluated by computing the trajectory of the ball. The algorithm can be found in [16]. In figure 3, the heights, H, reached by the bouncing ball ($\varepsilon = 0.83$) during 500 jumps have been plotted as a function of the acceleration, Γ_v. Each red point corresponds to one jump. The continuous line is the envelope curve that delimits the maximum height reached by the bead along 500 successive jumps. We may assume that the probability of escape is different from zero as soon as the centre of mass of the bead is beyond 30 mm. According to this envelope curve, the bead is allowed to escape when the height reached is higher than the so-called escape height, h_e, given by the height of a compartment, i.e. beyond $h_e = 30$ mm. The horizontal dashed lines represent the height of the compartment and the crossover height h_e. Consequently, at best, the beads may escape the compartment at an Γ_v of around 2 when shaken at 25 Hz. Note that these limits are held for one single bead in a compartment and that the interaction with the borders of the compartment is not taken into account. These facts may explain that we observe a few scarce exchanges below $\Gamma_v = 2$.

The intermediate state, i.e. when the beads are able to escape a trap but when large clusters are not observed, is difficult to interpret. This regime is located between $\Gamma_v = 1.5$ and 2. Even after 600 s, we cannot claim that a stationary state is reached. Most of the beads are located in a compartment with another bead. That may suggest that when n is larger than 2, the particles remain trapped. On the other hand, the dynamics change drastically above $\Gamma_v \approx 2$. The beads may then easily jump from one compartment to another and clusters form rapidly.

In figure 2 (bottom), the proportion P_{cluster} of the beads located in a compartment containing at least nine beads is represented with respect to the acceleration. This proportion is given by

$$P_{\text{cluster}} = \frac{1}{\mathcal{N}} \sum_{i \geqslant 9} \mathrm{i} C_i. \tag{1}$$

The proportion P_{cluster} is found to be null between $\Gamma_v = 0$ and $\Gamma_v = 2$ for the considered observation time. However, as explained in figure 3, above $\Gamma_v = 2$ the beads may escape from the compartment. A vertical line has been plotted in figure 2 (bottom) to visualize the threshold. Indeed, beyond this threshold, beads gather in clusters and consequently P_{cluster} strongly increases. At $\Gamma_v = 2.2$, up to 80% of the beads are in a cluster. The proportion of beads in the cluster decreases sharply to zero above $\Gamma_v = 3.5$. The experimental results show that between $\Gamma_v = 1$ (when the beads start bouncing) and $\Gamma_v = 1.5$, no large clusters can be observed. On the other hand, when the acceleration is above a certain threshold, large clusters are observed, also within a narrow window of acceleration between 2.2 and 3.5. These values depend on the frequency of the excitation, the coefficient of restitution of the beads and the height of the compartment walls.

A transition is observed between $\Gamma_v = 3.5$ and $\Gamma_v = 3.8$. The beads gather in some of the compartments when the acceleration is below $\Gamma_v = 3.5$. On the other hand, above this acceleration, the beads are re-distributed among the compartments. At high accelerations, the beads can easily jump out of a compartment. If the probability for a bead to be in a given compartment is random, we may think that the distribution of the beads in the compartments is a binomial one. Indeed, we may assume that one bead has a probability $1/N_c$ of landing in a compartment. As there are $\mathcal{N}$ beads, the probability of finding n beads in a compartment corresponds to the probability of observing n success out of $\mathcal{N}$ attempts with a probability $1/N_c$ of success. The distribution reads

$$P_n = \binom{\mathcal{N}}{n} (1/N_c)^n (1 - 1/N_c)^{(\mathcal{N}-n)}, \tag{2}$$

which is compared (blue crosses) to the distribution obtained experimentally with $\Gamma_v = 3.8$ in figure 2 (bottom). Good agreement is found between the model and the experiments. Note that the distribution may be approximated by a Poisson distribution.

In order to define the trapping, the escape time, $\tau_e(n)$, of a bead out of a compartment containing n beads has been evaluated. For this purpose, n beads are placed in a compartment. A cap is placed above the compartment to avoid any escape and the studied acceleration, Γ_v, is fixed. The cap is removed and a clock is started. The stop of the clock corresponds to the first escape of any beads. The operation is repeated 50 times in order to plot the cumulative distribution function $F(\tau_e(n))$. The escape direction is isotropic. However, it is recommended that the horizontality of the plate is checked carefully. In figure 4 (top), the cumulate distribution function (CDF) of the escape time, τ_e, is represented for n =1–9 in a semi-log plot (see the legend). The CDF is preferred to the probability distribution function because the results are independent of any arbitrary choice for the bin size. The solid lines represent fits with a Weibull law:

$$F(\tau_e(n)) = 1 - \exp(-\tau_e(n)/\lambda_n)^{k_n}, \tag{3}$$

where λ_n and k_n are the free fitting parameters. From the fits, when $n \geqslant 5$, the coefficient k_n is found close to unity. That means that the escape time behaves like an exponential distribution. The probability of escape does not depend on time; it looks like a Poissonian process. For small n, k_n is found to range between 1.5 and 2. This means that the escape probability

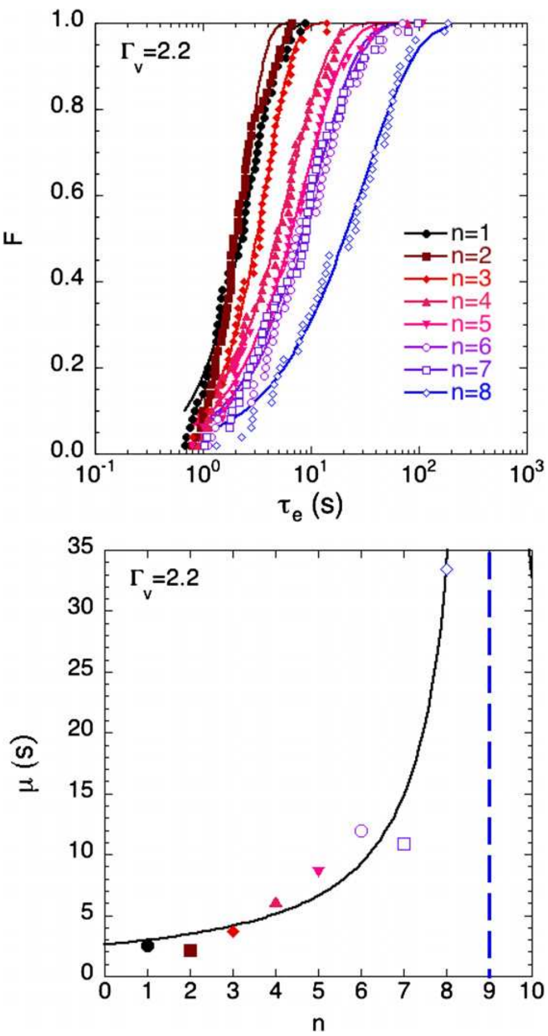

Figure 4. Top: cumulative distribution function F of escape times τ_e of one bead out of a compartment containing n beads when $\Gamma_v = 2.2$ (see the legend). Bottom: mean value of τ_e from equation (4) as a function of the number of beads contained in a compartment. The curve is a fit using equation (5). Note that the symbols used for the top and bottom figures are the same.

increases with time. This should be interpreted as an ageing process. In order to compare the escape time, the mean value, μ, of the escape time is plotted versus the number of beads, n, in a compartment in figure 4 (bottom). This quantity is given as follows:

$$\mu = \lambda_n \Gamma\left(1 + \frac{1}{k_n}\right), \tag{4}$$

where Γ is the gamma function. The escape time becomes very difficult to measure for $n = 9$ as the escape time is very large. Hou *et al* [11] have characterized the oscillation time by a power that diverges for a critical maximum speed of the plate (that corresponds to a maximum acceleration of the plate). Similarly, the dependence of μ on n has been fitted by a divergence power law,

$$\mu \propto |n - 9|^{\beta}, \tag{5}$$

where β is a fit parameter and is found to be -1.15. Strictly speaking, a divergent function is not appropriate in our case as, beyond $n = 9$, the probability of escape is very small but cannot be considered as zero.

The threshold number (here $n = 9$) to obtain a cluster depends on the geometry of the compartment and on the bead–bead collision rate. Indeed, the cluster mechanism is related to the energy dissipation during the numerous bead–bead shocks. Consequently, the more the beads shock each other, the more clusters may form. There must exist a link between the threshold number and the mean free path of the particles. The mean free path, ℓ, may be evaluated as a function of the number of beads contained in the compartment. The simplest way is to consider that each bead occupies the same space in the compartment: we divide the volume of the compartment by the number of beads contained in the compartment. In doing so, the mean distance between two particles is proportional to $n^{-1/3}$. The number of shocks per second is obtained by knowing the mean speed of the beads. The problem becomes rather complex because the distribution of speed is unknown. Moreover, the collective motion of the beads contained in a compartment must be studied as a non-dilute hard sphere gas which is definitely not trivial.

To sum up the experimental results, three phases can be observed: frozen, clustering and binomial gas. The clustering occurs in a narrow range of accelerations. In that range, the compartment becomes a strong trap centre when more than eight beads are located in it. The next section proposes to model the trapping strength of a site and to simulate the clustering process for a large number of sites. The experiments are limited by the number of compartments and by the influence of the side walls surrounding the whole grid. Even by reproducing the experiments, it is quite difficult to obtain the distribution of the compartment population when clusters occur since there are four to five clusters per experiment. We develop a simple model in order to investigate the distribution of clusters in a large periodic system.

4. Network simulation

The simulation is a very simple application for describing the motion of the bead. The grid is represented by a 512×512 matrix to which periodical conditions are applied. The value of a matrix cell is characterized by the number of particles in that case. At each step, one grain of each cell is asked to move randomly towards the north, south, east or west. The number of steps has been fixed to 10^5 (stationary configurations are obtained). The escape probability, $P_{\text{out}}(n)$, is guided by a distribution function that depends on the number of beads in the site and is directly inspired by the experimental results (figure 5). In doing so, the acceleration is only implicitly taken into account as $P_{\text{out}}(n)$ is a function of Γ_v. We consider here the case for which clustering is present. It should be noted that the important feature of the function $P_{\text{out}}(n)$ is to provide a strong contrast of the escape probability beyond a given number n. The escape probability is equal to 1 when $n = 1$ and then decreases rapidly to a value of 3×10^{-3} when $n = 9$. We found that the maximum number of beads that a compartment can contain is about 70. We arbitrarily decided to have a symmetric function about the number 35. Consequently, the probability remains constant between 9 and 63. Above $n = 63$, P_{out} increases towards 1 at $n = 70$ (corresponding to a full compartment) to take into account the fact that the number of beads is limited in a compartment.

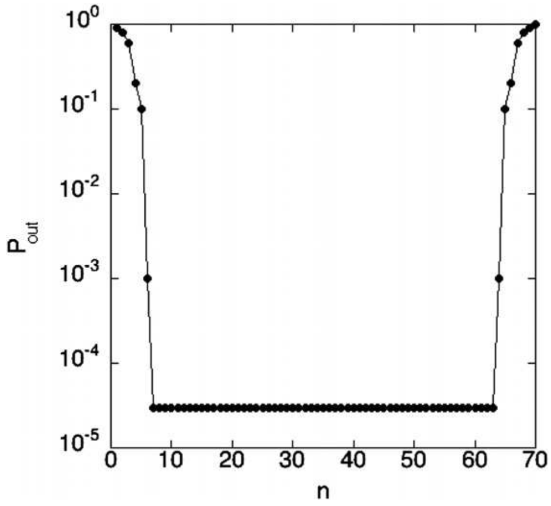

Figure 5. Model for the escape probability P_{out} with respect to the number, n, of beads contained in the compartment inspired by the experimental data.

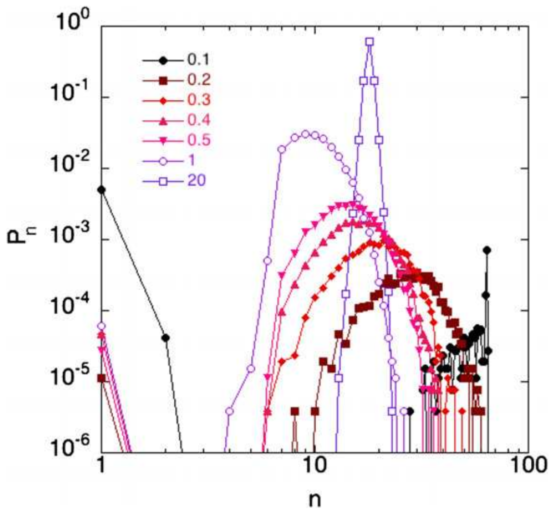

Figure 6. Probability P_n of finding a compartment containing n beads. The legend indicates the filling factor η.

Several initial conditions have been tested. We choose to tune the filling factor, $\eta = \mathcal{N}/N_c$, between 0.1 and 20. The beads are uniformly distributed. For example, for the initialization of the number of beads per site and for $\eta = 0.1$, a compartment has one chance in ten to be populated by one bead. Figure 6 compares the probability, P_n, of finding a compartment with n beads for different filling factors η. For any filling fraction lower than 9, the vast majority of the compartments contained zero particles (not represented in the log–log plot). For the highest volume fraction shown, most of the beads remain in their initial compartment as the escape probability is very low for such a highly populated compartment. When the system is diluted ($\eta = 0.1$), compartments containing more than 35 beads can be found. In that situation, as soon as a site contained more than nine beads, it grows and continues to grow. As there are few of them, they 'pump' the particles up to be completely full. For η in the range 0.2–1, the distribution of P_n evolves towards an increase of compartments populated by between 9 and 20 beads. The number of traps is larger than in dilute systems. Consequently, they rapidly pump all the beads in their direct vicinity and can no longer grow as the system is jammed in the numerous traps.

5. Conclusion

The exchange of particles between a periodical network of traps has been studied. According to the energy provided by the plate (here through the control of the maximum acceleration, Γ_v), several stationary states are observed. Starting from low forcing accelerations of the plate: (i) the particles remain in their initial compartment (frozen state), (ii) the particles jump from one compartment to another and gather in some of the compartments (cluster state) and (iii) the particles may jump out of the compartment whatever the number of beads ('binomial' gas state). In order to better describe the cluster state, the escape probability has been measured with respect to the initial number, n, of beads in a compartment. A strong decrease of the escape probability has been observed for a given number of beads (here $n = 9$). A simple numerical model has been built in order to study the influence of the filling fraction on the clustering.

This problem is particularly adapted to undergraduate students to assist a lecture on statistical physics. Several concepts are applied such as distribution functions, particular gasses, mean free paths, dissipative systems and bouncing ball nonlinearities.

Complementary experiments should be performed by measuring the influence of the initial number of beads per compartment as suggested by the simulations. Moreover, the escape time dependence with the acceleration may also be measured for other geometries, e.g. a larger ratio between the grain size and compartment height.

Acknowledgments

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